

The Art of Solving Problems

A practical method for thinking under a clock, with the SAT as the training ground.

0	The Top of the Scale Is a Thinking Test	5 minutes
1	Understand: Read What Is Actually There	8 minutes
2	Represent: Choose How to See the Problem	8 minutes
3	Strategize: The Heuristic Toolkit	12 minutes
4	Decide: Choosing Among Answers	8 minutes
5	Execute and Check: Reliability	9 minutes
6	Tools You Should Not Have to Think About	8 minutes
7	Training to 1600	6 minutes
8	Practice Set	about 30 minutes to solve, 15 to review

By Lamai Hons, for Margin at marginsat.com. Original practice questions, not affiliated with College Board.

The Top of the Scale Is a Thinking Test

5 minutes Goal: see where top scorers actually lose points

0.1 The claim

Below about 1400, most lost points come from content a student has not learned yet. Above about 1500, that stops being true. A strong student's misses come mostly from four things: reading the question slightly wrong, choosing an answer that was designed to look right, making a small slip while executing, and freezing on a problem whose first move they did not see.

More content review does not fix any of these. A method does: how you read, how you plan, how you choose between answers, and how you check. Mathematicians call that method *problem solving*. It is a skill you can learn on purpose, and it carries over to every exam, contest and technical job you will ever have. This packet teaches it, using the SAT as the training ground.

0.2 The test in one table

Section	Modules	Questions each	Time each	Average per question
Reading and Writing	2	27 (25 scored, 2 unscored)	32 min	about 71 seconds
Math	2	22 (20 scored, 2 unscored)	35 min	about 95 seconds

- **Adaptive.** Module 1 mixes easy, medium and hard questions. How you do on it decides whether Module 2 is harder or easier. The top scores are only reachable through the harder Module 2, so Module 1 accuracy matters more than Module 1 speed.
- **Reading and Writing:** passages of 25 to 150 words, one question per passage. Questions that test similar skills are grouped together and run from easiest to hardest.
- **Math:** the built-in Desmos calculator is allowed on every question. About 75% of questions are multiple choice and 25% ask you to type your own answer. A reference sheet is always available.
- **No penalty for wrong answers.** The 2 unscored questions per module look like the others; you cannot tell which they are, so treat every question as real.
- There is a 10-minute break between the two sections.

0.3 The error budget

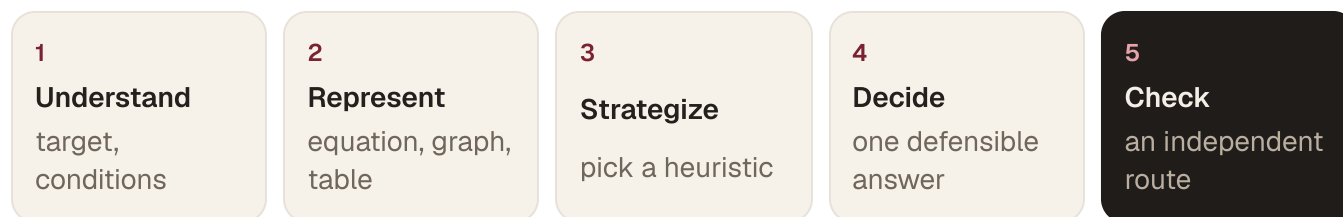
There are 98 questions. To land reliably at or near 1600, you can afford almost no misses, and each miss near the top costs more than a miss in the middle of the scale. Put another way, your error rate per question has to be around 1 to 2 percent.

For a student who already knows the material, getting from 5 percent to 2 percent is not about knowing more. It is about doing the same things more reliably. A 1600 is a **reliability score**.

Source of the miss	What happened	Fixed in
Misread	You answered a slightly different question than the one asked.	Part 1
Stuck	You did not see how to start, or chose a slow route.	Parts 2 and 3
Trap	You picked a wrong answer built to attract exactly your reasoning.	Part 4
Slip	Arithmetic, sign, copying, or clicking error.	Part 5
Gap	You did not know a fact, rule or word.	Part 6

KEY IDEA How to read this packet

Parts 1 to 5 follow the order in which you actually solve a problem: understand it, represent it, find a strategy, decide among answers, then execute and check. Part 6 is the stock of facts a problem solver should not have to think about. Part 7 is a training plan. Part 8 is a practice set; each problem is tagged with the idea it practices.



If the check fails, go back to step 1, not step 3.

Understand: Read What Is Actually There

8 minutes Goal: never answer a question you were not asked

1.1 Pólya's four phases

In *How to Solve It* (1945), the mathematician George Pólya split problem solving into four phases: **understand** the problem, **devise a plan**, **carry out** the plan, and **look back**. Students spend nearly all their time in phases 2 and 3, because those feel like work. At the top of the SAT, most errors happen in phases 1 and 4, which feel like they are not doing anything. This part is about phase 1.

1.2 Find the target

Before solving anything, answer one question: *what exactly must my final answer be?* Not “ x ” in general, but “the value of $3x - 1$ ”, “the number of minutes”, “the choice that most directly *weakens* the claim”. Write the target on your scratch paper and box it. When you finish, compare your answer with the box.

EXAMPLE the target is not the variable

Problem. If $4x - 6 = 18$, what is the value of $2x - 3$?

Solution. The target is $2x - 3$, which is exactly half of $4x - 6$. So $2x - 3 = 18/2 = 9$. Solving for x first gives $x = 6$, and 6 is almost certainly one of the answer choices. It is the most common wrong answer, because the student solved the problem they expected instead of the one on the screen.

1.3 Inventory the conditions

List every condition the problem gives you: numbers and their units, restrictions such as *positive*, *integer*, $x > 0$, *for all x* , and logical words such as *not*, *except*, *approximately*, *in terms of*. On a well-written test, every condition is there for a reason. If you finish without using one, either you got lucky or you made an error.

EXAMPLE the condition carries the problem

Problem. The graph of $f(x) = x^2 - 6x + c$ has exactly one x -intercept. What is the value of c ?

Solution. The whole problem lives in the phrase *exactly one*. A parabola touches the x -axis exactly once when its vertex sits on the axis. The vertex is at $x = -\frac{-6}{2} = 3$, and $f(3) = 9 - 18 + c = 0$, so $c = 9$. A second route, useful as a check: one x -intercept means the discriminant $b^2 - 4ac$ is zero, so $36 - 4c = 0$ and $c = 9$.

1.4 Restate in plain words

If you cannot restate a question in ten words or fewer, you do not understand it yet. “Which choice best describes the function of the underlined sentence in the text as a whole?” becomes *why is this sentence here?* “Which finding, if true, would most directly undermine the researchers’ hypothesis?” becomes *which fact makes their idea less likely?* The plain version is what you test each choice against.

1.5 Read at the right resolution

Most of the logic in a question lives in small words: *not, only, all, some, most, first, may, must, more than, less than, unless, although, however, even*. Skimming past one of them turns a right method into a wrong answer. In both sections, highlight (or underline on paper) the logic words in the question before you read the choices. It takes two seconds and removes a whole class of errors.

TRAP Direction flips

Some questions ask for the opposite of the usual direction: *weaken* instead of support, *least* instead of most, *which is NOT true*, *decrease* instead of increase, *challenge* instead of illustrate. Your brain is trained on the usual direction and will quietly correct the question back to it. Defense: before you choose, say the direction to yourself in plain words, such as “I need the fact that *hurts* the claim.” Then check the choice against that sentence, not against the passage in general.

1.6 Read the passage for its logic, not its topic

Each Reading and Writing passage is a small argument. Its sentences play roles: background, claim, evidence, qualification, counterpoint, conclusion. The question is almost always about those roles and how they connect, not about the subject matter. Two habits help:

- **Label roles as you read.** A one- or two-word label per sentence (“old view”, “but”, “evidence”, “so”) is enough.
- **Leave your own knowledge outside.** If the passage mentions a place you have been or a poem you studied, notice the pull and set it aside. Familiar topics feel easy, and that feeling is exactly what makes students skip the details. The right answer must come from the text in front of you.

EXAMPLE main idea versus true detail

Many gardeners assume that earthworms always improve soil. In most farmland, the assumption holds: worm tunnels let water and air reach roots, and worm castings are rich in nutrients. But some northern hardwood forests developed without native earthworms after the last ice age, and in these forests introduced European worms consume the thick layer of fallen leaves that native wildflowers and tree seedlings depend on. Ecologists have documented sharp declines in these plants where the worms have spread.

Problem. Which choice best states the main idea of the text?

- A. Earthworms improve farmland soil by letting water and air reach plant roots.
- B. Although earthworms benefit many soils, introduced earthworms can harm forests that developed without them.
- C. Native wildflowers in northern forests depend on earthworms to break down fallen leaves.
- D. Gardeners should remove earthworms from their soil.

Solution. Label the roles: sentence 1 is the common view, sentence 2 grants it in one setting, sentences 3 and 4 (“But...”) give the exception and its evidence. The main idea joins the concession and the exception: **(B)**. Choice (A) is true but is only the conceded detail. Choice (C) reverses the passage: the plants depend on the leaves that the worms destroy. Choice (D) is advice the text never gives.

TOOLKIT Phase 1 checklist

Target. What exactly is the answer: which quantity, which units, which direction?

Conditions. What restrictions and logic words are there? Have I used them all?

Plain words. Can I restate the question in ten words?

Answer shape. What should the answer look like? A positive number? Roughly how big? A contrast word? A claim that matches the passage’s caution?

Familiar? Have I seen a problem with the same target or the same structure? (Section 3.12.)

USE WHEN always, on every question. It takes 5 to 10 seconds and prevents the most expensive mistakes on the test.

Represent: Choose How to See the Problem

8 minutes Goal: turn a problem into the form where it is easy

2.1 Why representation matters

The same problem can be hard as a paragraph and easy as an equation, hard as an equation and easy as a graph, hard as a graph and easy as a table. Strong solvers do not have better algebra; they switch representations faster. The main ones are:

- **Symbols:** equations and expressions. Best for exact answers and for structure.
- **Graphs:** best for counting solutions, intersections, maximums and minimums, and anything with a parameter.
- **Numbers:** a table of values or a test case. Best for spotting patterns and checking claims.
- **Pictures:** a labeled diagram. Best for geometry and for any word problem with positions, distances or overlaps.
- **Plain words:** a one-line restatement. Best for Reading and Writing, and for checking that the math matches the story.

2.2 Words to symbols: translation

Word problems are translation exercises. Translate one clause at a time and define every variable with its units (“let t be the number of hours after 9 a.m.”), not just “let t = time”.

Phrase	Symbols	Phrase	Symbols
a is 5 more than b	$a = b + 5$	a is 5 less than b	$a = b - 5$ (order flips)
p percent of x	$\frac{p}{100}x$	what percent of A is B	$\frac{B}{A} \times 100$
increased by $p\%$	$\times \left(1 + \frac{p}{100}\right)$	decreased by $p\%$	$\times \left(1 - \frac{p}{100}\right)$
per, each, for every	a rate: divide	twice as many a as b	$a = 2b$

EXAMPLE the order trap in “less than”

Problem. The number of members in Club A is 12 less than twice the number of members in Club B. Together, the clubs have 93 members. How many members does Club B have?

Solution. Let a and b be the numbers of members. “12 less than twice b ” is $2b - 12$, not $12 - 2b$. So $a = 2b - 12$ and $a + b = 93$. Substituting, $3b - 12 = 93$, so $b = 35$ (and $a = 58$). Check against the story: 58 is 12 less than 70, and $58 + 35 = 93$.

2.3 Graphs: Desmos as a way of seeing

The built-in Desmos calculator is a representation tool, not just a calculator. Reach for it when a question is about *how many* solutions there are, *where* two things meet, a *maximum or minimum*, or a *parameter* like k .

- **Solutions of $f(x) = g(x)$:** graph $y = f(x)$ and $y = g(x)$ and count or click the intersections.
- **Parameters:** type the equation with k in it; Desmos offers a slider. Drag it and watch for the moment the condition in the problem happens.
- **Equivalent expressions:** graph both. If the curves coincide everywhere, the expressions are equivalent.
- **Regression:** enter data in a table, then type $y1 \sim m x1 + b$ (or $a b^{x1}$ for exponential).
- **Vertex, zeros, intercepts:** click the graph; Desmos marks these points.

Two limits. Desmos gives decimals, so confirm exact values (for example, $0.333 \dots$ should become $\frac{1}{3}$). And some questions are built so that graphing is slower than thinking, especially when several letters are unknown. Use whichever route is faster, and when time allows, use the other one as your check.

EXAMPLE a parameter question, solved two ways

Problem. For what value of k does the line $y = 2x + k$ intersect the parabola $y = x^2$ at exactly one point?

Symbols. Set them equal: $x^2 = 2x + k$, so $x^2 - 2x - k = 0$. Exactly one intersection means exactly one solution, so the discriminant is zero: $(-2)^2 - 4(1)(-k) = 4 + 4k = 0$, giving $k = -1$.

Graph. Graph $y = x^2$ and $y = 2x + k$ with a slider for k . As k decreases, the line slides down and touches the parabola at a single point when $k = -1$, at $(1, 1)$.

This pattern (*line meets curve exactly once, no solution, infinitely many solutions*) appears often on hard Module 2 questions. Section 3.5 has the rule for each version.

2.4 Tables: organizing information

Tables turn a tangle of conditions into rows and columns. The most important use on the SAT is conditional probability: the phrase *given that* or *selected from those who...* restricts the denominator to one row or one column.

EXAMPLE the denominator trap

A school surveyed 200 students.

	Grade 10	Grade 11	Total
Plays a sport	36	54	90
Does not play a sport	64	46	110
Total	100	100	200

Problem. If a student who plays a sport is selected at random, what is the probability that the student is in grade 11?

Solution. “A student who plays a sport” means the selection is only from the 90 students in that row. So the probability is $\frac{54}{90} = 0.6$. Three tempting wrong answers come from using the wrong denominator: $\frac{54}{200} = 0.27$ (the whole survey), $\frac{54}{100} = 0.54$ (the grade 11 column), and $\frac{54}{110}$ (the wrong row). The table makes the right denominator visible.

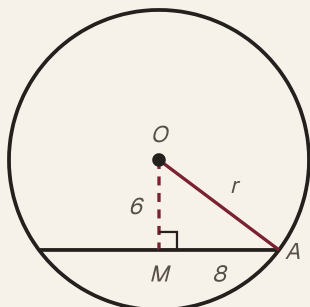
2.5 Pictures: draw it, label it, add a line

Draw every geometry figure on your scratch paper, even when one is given, and label all the lengths and angles you know. Then look for the one extra line that creates a right triangle or a pair of similar triangles. The most useful extra lines: a radius to a point of tangency (it is perpendicular to the tangent line), a perpendicular from the center of a circle to a chord (it cuts the chord in half), and a height dropped inside a triangle.

EXAMPLE one extra line

Problem. In a circle, a chord of length 16 lies 6 units from the center. What is the radius of the circle?

Solution. Draw the perpendicular from the center O to the chord. It meets the chord at its midpoint M , so $AM = 8$. Triangle OMA has a right angle at M , so $r^2 = 8^2 + 6^2 = 100$ and $r = 10$. (It is the 6-8-10 triangle, a multiple of 3-4-5.)



2.6 Representing a passage

Reading and Writing questions have representations too:

- **Role map:** one word per sentence (“claim”, “but”, “evidence”). Use it for structure, purpose, main idea and inference questions.
- **Two one-liners:** for questions with two texts, write each author’s position in one line, then find the exact point where they touch. Authors rarely disagree about everything.
- **Goal line:** for questions that give you bullet-point notes, the goal in the question *is* the problem. Write it down in two or three words (“contrast the two banks”) and ignore the notes until you need them.
- **Skeleton:** for grammar questions, strip the sentence to subject, main verb and connectors. Most boundary and agreement errors are obvious once the phrases in the middle are crossed out.

Strategize: The Heuristic Toolkit

12 minutes Goal: always have a next move, even when you do not see the whole path

3.1 What a heuristic is

A **heuristic** is a rule of thumb that suggests a next move when you do not see the solution. It is not guaranteed to work; it is guaranteed to get you moving. The eleven below are adapted from Pólya and from contest problem solving, and each one is shown on SAT-style problems. When you are stuck, run down the list in Section 3.13 and ask “does this one apply?” That is what experienced solvers do, only faster.

3.2 Look for structure before you compute

Ask what the expression *is*, not just what it equals. Many questions are built so that the target is a combination of the givens and no individual variable needs to be found.

EXAMPLE combine, do not solve

Problem. If $(x + y)^2 = 49$ and $xy = 6$, what is the value of $x^2 + y^2$?

Solution. $(x + y)^2 = x^2 + 2xy + y^2$, so $x^2 + y^2 = 49 - 2(6) = 37$. Solving for x and y separately is possible but slow and error-prone.

EXAMPLE add the equations

Problem. If $3a + 2b = 17$ and $a - 2b = -1$, what is the value of $4a$?

Solution. Adding the equations cancels b : $4a = 16$. Done in one line. Before solving any system, check whether adding or subtracting the equations gives the target directly.

USE WHEN the question asks for an expression such as $x^2 + y^2$, $A + B$ or $Y - X$, rather than a single variable.

3.3 Specialize: plug in your own numbers

When the answer choices contain variables, or the question says *must be true* or *for all values*, replace the letters with simple numbers you choose, compute the answer, and test each choice. Rules: avoid 0 and 1 (they make different expressions look equal), avoid numbers that appear in the problem, and if two choices survive, try a second set of numbers.

EXAMPLE percent changes with no prices given

Problem. A store raises the price of a jacket by 20%, and later lowers the new price by 25%. The final price is what percent of the original price?

Solution. Let the original price be \$100. After the increase it is \$120; 25% of 120 is 30, so the final price is \$90, which is **90%** of the original. The trap answer is 95% (“up 20, down 25, net down 5”). Percent changes multiply: $1.20 \times 0.75 = 0.90$.

USE WHEN variables in the choices; “must be true”; percent problems without a starting amount; “in terms of” questions.

3.4 Work backward from the choices

When the choices are numbers and the forward algebra looks messy, test the choices in the original conditions. Numeric choices usually appear in increasing order, so start with one in the middle; if it is too big or too small, you know which way to move.

EXAMPLE test the choices

Problem. The square of a number exceeds 4 times the number by 45. Which of the following could be the number?

- A. -9
- B. -5
- C. 5
- D. 15

Solution. Test: for -5 , $(-5)^2 = 25$ and $4(-5) = -20$; $25 - (-20) = 45$. Yes, **(B)**. (Forward: $x^2 - 4x - 45 = 0$ factors as $(x - 9)(x + 5) = 0$, so $x = 9$ or $x = -5$; only -5 is offered.)

In Reading and Writing, working backward means testing each choice against your plain-word version of the question, one at a time, and writing down why each fails.

USE WHEN numeric choices and slow algebra; “which of the following is a solution”; any time you are stuck and there are only four options.

3.5 Go back to definitions

Many hard questions are just definitions in disguise. Translate each phrase into what it literally means, then substitute.

The problem says	It means
$x = a$ is a solution of $p(x) = 0$	$p(a) = 0$: substitute and solve for the unknown constant.
$(x - a)$ is a factor of $p(x)$	$p(a) = 0$ (the factor theorem). The remainder when $p(x)$ is divided by $x - a$ is $p(a)$.
The graph passes through (a, b)	$f(a) = b$.
A linear system has no solution	The lines are parallel: same slope, different intercepts.
A linear system has infinitely many solutions	The two equations describe the same line: one is a multiple of the other.
A quadratic has exactly one real solution	Discriminant $b^2 - 4ac = 0$ (vertex on the axis).
A quadratic has no real solutions	$b^2 - 4ac < 0$.
A line is tangent to a curve	Substituting gives a quadratic with discriminant 0.

EXAMPLE the factor theorem

Problem. If $x - 3$ is a factor of $p(x) = 2x^3 - kx + 6$, what is the value of k ?

Solution. Being a factor means $p(3) = 0$: $54 - 3k + 6 = 0$, so $k = 20$. No division needed.

EXAMPLE no solution

Problem. For what value of c does the system $6x - 4y = 9$ and $cx - 2y = 5$ have no solution?

Solution. No solution means parallel lines: the x and y coefficients must be in the same ratio, but the constants must not. The y -coefficients have ratio $-4 : -2 = 2 : 1$, so we need $6 : c = 2 : 1$, giving $c = 3$. Check the constants: $9 : 5 \neq 2 : 1$, so the lines are parallel and distinct, not identical.

In Reading and Writing, the same move works on words. If you are unsure what a choice claims, define its key word precisely. “Undermine” means make less likely, not disprove. “Qualify” means limit, not reject.

USE WHEN a phrase like “is a factor”, “is a solution”, “passes through”, “no solution”, “exactly one”, “tangent”.

3.6 Find what does not change

When something changes in a problem, look for a quantity that stays fixed. That fixed quantity is usually the key equation.

EXAMPLE mixtures: the dissolved amount is fixed

Problem. A 40-liter salt solution is 15% salt. How many liters of water must evaporate for the solution to become 20% salt?

Solution. Evaporation removes water but no salt. Salt: $0.15 \times 40 = 6$ liters, before and after. After evaporation, 6 liters is 20% of the total, so the total is $6/0.20 = 30$ liters. So $40 - 30 = 10$ liters evaporate.

Other quantities that typically stay fixed: the total in a ratio problem where items move between groups, the distance in a round trip, the area when a shape is cut and rearranged, the product in an inverse variation ($xy = k$).

USE WHEN mixtures, dilution, evaporation; items moving between groups; “before and after” problems.

3.7 Look at the extremes

The largest, smallest, first or last case often has extra structure. A quadratic reaches its maximum or minimum at its vertex. A “could be” question needs only one case; a “must be” question can be killed by one extreme counterexample.

EXAMPLE maximum at the vertex

Problem. The height, in feet, of a ball t seconds after it is thrown is $h(t) = -16t^2 + 64t + 5$. What is the maximum height of the ball?

Solution. The vertex is at $t = -\frac{64}{2(-16)} = 2$, and $h(2) = -64 + 128 + 5 = 69$ feet. Check with Desmos by clicking the top of the parabola.

In Reading and Writing, check how strong each answer is. A choice that says “proves”, “all” or “always” needs a passage that is just as strong. If the passage says “suggests” or “in some cases”, the right answer will be equally careful.

USE WHEN maximum, minimum, at least, at most, could be, must be.

3.8 Find a pattern, or solve a smaller problem

If a problem is too big, try a smaller version, or compute the first few cases and look for the pattern. On the SAT, the most common pattern question is deciding what kind of function a table describes.

EXAMPLE linear or exponential?

x	0	1	2	3
y	5	15	45	135

Solution. The differences are 10, 30, 90, not constant, so the function is not linear. The ratios are all 3, so it is exponential: $y = 5 \cdot 3^x$. **Rule:** equal differences mean linear; equal ratios mean exponential.

USE WHEN tables, sequences, “which function could model”, and any question you cannot start.

3.9 Decompose

Break a problem into pieces you can solve. Split a strange shape into rectangles and triangles. Turn a long word problem into a chain of short steps, each with a labeled result. Strip a long sentence down to its subject and main verb before judging its grammar.

EXAMPLE strip the sentence

Problem. *The collection of letters written by the explorer during her three expeditions now housed in the university's archive.*

- A. are
- B. is
- C. were
- D. have been

Solution. Cross out the phrases in the middle: *The collection* [of letters written by the explorer during her three expeditions] now housed. The subject is *collection*, which is singular, so the answer is **(B)** *is*. The plural nouns *letters* and *expeditions* sit next to the blank to pull you toward *are*.

USE WHEN compound figures, multi-step word problems, long sentences, two-part answer choices (check each half separately).

3.10 Change your point of view

If the direct route is hard, look at the problem from the other side. For an area, subtract the unshaded part from the whole. For “at least one”, compute one minus the probability of “none”. In Reading and Writing, if no choice clearly fits, flip the task: find the specific reason each wrong choice fails. The survivor is your answer, and you will know why.

USE WHEN shaded regions, “at least”, “not”, and multiple-choice questions where nothing jumps out.

3.11 Estimate and eliminate

Before computing, estimate the size and sign of the answer, then eliminate choices that cannot be right. A rough estimate often removes two or three choices in seconds, and it is also a powerful check after you compute. If the answer must be a little less than 30 and you got 300, you know to look again.

EXAMPLE estimation kills the trap

Problem. A population is modeled by $P(t) = 800(1.5)^{t/4}$, where t is in years. By about what percent does the population increase every 2 years?

- A. 12.5%
- B. 22.5%
- C. 25%
- D. 50%

Solution. Every 4 years the population grows by 50%. Two periods of 2 years must compound to 50%, so each one is less than half of 50%: $(1 + r)^2 = 1.5$ means $r < 25\%$. That eliminates (C) and (D) immediately. Exactly: $1.5^{1/2} \approx 1.2247$, so the increase is about **22.5%**, which is (B). The trap (C) assumes growth adds up instead of multiplying.

USE WHEN always as a check; first when the arithmetic is heavy or the choices are far apart.

3.12 Use a problem you have solved before

Pólya’s most useful question is “*Do you know a related problem?*” Most SAT questions are new versions of a small number of patterns. The more patterns you can recognize, the less you have to invent under time pressure. Build the library on purpose: every time you miss a question or solve one slowly, write down the pattern and its first move (Part 7 shows how). After a few hundred questions, “new” hard problems start to look familiar.

3.13 Signals: which heuristic to try first

If the problem...	Try first
asks for an expression, not a variable	structure (3.2)
has variables in the answer choices; says “must be true”	plug in numbers (3.3)
has numeric choices and messy algebra	work backward (3.4)
says no solution / infinitely many / exactly one / tangent	definitions table (3.5), or a Desmos slider
says “is a factor”, “is a solution”, “passes through”	substitute the point (3.5)
involves mixtures, dilution, items moving between groups	find the fixed quantity (3.6)
asks for a maximum, minimum, “could be” or “must be”	extremes, vertex (3.7)
gives a table of values or a sequence	differences and ratios (3.8)
has a compound figure, many steps, or a long sentence	decompose (3.9)
has a shaded region, “at least one”, or “not”	change point of view (3.10)
has heavy arithmetic or widely spread choices	estimate first (3.11)
says “given that” or “selected from those who”	restrict the denominator (2.4)
involves a circle with a chord or tangent	add a radius or perpendicular (2.5)
is a Reading and Writing “best supports / weakens”	state the claim and direction, then test each choice (Part 4)
gives bullet-point notes	goal first (6.4)

Decide: Choosing Among Answers

8 minutes Goal: pick the answer you can defend, not the one that feels best

4.1 The one-defensible-answer principle

Every SAT question is written so that exactly one choice can be defended from the text or the math, and each of the other three is wrong for a specific reason you could name. Your job is not to find the most attractive answer. It is to find the one you could defend to a skeptical grader, with a line of the passage or a line of algebra. “It sounds right” is not a defense. If you cannot say why an answer is right, you have not finished the question.

4.2 Predict before you look

The wrong choices are written to pull you toward them. The defense is to have your own answer before you read them: a number, a word for a blank, a one-line claim, a relationship (“contrast”, “cause”). Then read the choices looking for a match. A prediction does not need to be exact. A rough one, like “something meaning *cautious*” or “the line about the control group”, still protects you, because you are now checking choices against something instead of reacting to them.

4.3 Anatomy of a wrong answer

Wrong answers are not random. Test writers build them from a short list of designs, and once you know the list you can name the flaw in most wrong choices within seconds.

Design	How it works
Half right	The first half matches the passage, the second half does not. Read every word of a choice.
Too strong	Right direction, but stronger than the text allows: “proves”, “all”, “always”, “the only”, or a causal claim where the text shows only a link.
Out of scope	On topic and plausible, but not stated or implied anywhere in the text.
Reversed	Flips a relationship: cause and effect, support and weaken, increase and decrease, earlier and later, A causes B versus B causes A.
True but irrelevant	Accurate according to the passage or graph, but does not do what the question asks. Very common on data and notes questions.
Wrong subject	A correct statement about the wrong thing: the other author, the other group, the other column in the table.
Recycled words	Repeats striking words from the passage inside a claim the passage does not make.
Math: stopped early	An intermediate value: x when the question asked for $2x$, the radius when it asked for the area.
Math: common slip	The result of a typical error: a dropped negative, adding percents, using the wrong denominator, the wrong unit.

4.4 The two-filter test for evidence and data

On evidence and data questions, a correct choice must pass two filters:

1. **Accurate:** what it says is true according to the quotation, table or graph.
 2. **Relevant:** it supports (or weakens) the *specific* claim in the question, in the right direction.
- Most wrong answers pass one filter and fail the other. “True but irrelevant” is the most common trap at the top of the scale, because a student who checks only accuracy finds nothing wrong with it.

EXAMPLE two filters on a table

Agronomists testing a new nitrogen fertilizer found that it helped some crops more than others. Corn produced far more bushels per acre than wheat in every trial, but the agronomists concluded that the fertilizer's benefit was greater for wheat than for corn.

Crop	Average yield without fertilizer	Average yield with fertilizer
	(bushels per acre)	(bushels per acre)
Wheat	48	63
Corn	172	181
Soybeans	51	58

Problem. Which choice most effectively uses data from the table to support the agronomists' conclusion?

- A. Corn produced more bushels per acre than wheat, both with and without the fertilizer.
- B. With the fertilizer, the wheat yield rose by 15 bushels per acre, while the corn yield rose by only 9.
- C. With the fertilizer, the soybean yield rose by 7 bushels per acre, less than the increase for wheat.
- D. With the fertilizer, the corn yield rose by 15 bushels per acre, while the wheat yield rose by only 9.

Solution. The claim is "the benefit was greater for wheat than for corn". (A) is accurate but irrelevant: it is about total yield, not benefit. (C) is accurate but about the wrong subject. (D) reverses the crops. **(B)** is accurate ($63 - 48 = 15$ and $181 - 172 = 9$) and directly supports the claim.

4.5 When two answers both look right

Put them side by side and find the exact words where they differ. The decision is almost always made in that small difference, and usually one version is a little too strong, a little off-target, or about a slightly different subject. Ask which version the text actually supports. Never choose by which one sounds more sophisticated.

EXAMPLE strength decides

In a survey of 1,200 adults, researchers found that people who reported reading fiction regularly scored higher on a test of recognizing emotions in faces than people who did not. The researchers caution that the survey shows only an association: people who are already skilled at reading emotions may simply be more drawn to fiction.

Problem. Which choice best states the researchers' conclusion?

- A. Reading fiction improves people's ability to recognize emotions.
- B. Regular fiction reading is linked to skill at recognizing emotions, but the survey does not show which causes which.
- C. People who are skilled at recognizing emotions read more fiction because of that skill.
- D. Fiction readers and other adults performed about equally on the emotion test.

Solution. (D) contradicts the finding, so the real contest is among (A), (B) and (C). Compare them word by word. (A) claims fiction *causes* the skill. (C) claims the skill *causes* the reading. The researchers explicitly refuse to claim either direction. **(B)** matches the strength of the passage exactly. Both (A) and (C) are the "too strong" design, pointed in opposite directions.

4.6 Guessing is a decision too

There is no penalty for a wrong answer, so never leave a question blank. If you are stuck, eliminate what you can with stated reasons, pick from what remains, mark the question for review, and move on. Eliminating two choices turns a 25% guess into a 50% guess. For a typed-answer math question, enter your best estimate. A guess has some chance of being right; a blank has none.

4.7 Calibration: knowing what you know

As you work, sort each answer into one of three levels: **sure** (you can state the defense), **likely**, or **guess**. Mark every "likely" and "guess" for review. Then, in practice, keep score of your "sure" answers. If more than about 1 in 50 of them turn out wrong, your feeling of certainty is not reliable yet, and the fix is not to work harder but to change the rule: you are not allowed to feel sure until you have stated the defense.

If your scores regularly come in 20 or 30 points below what you expected, the gap is usually made of answers that felt certain and were never checked. Good calibration tells you where to spend your review time.

KEY IDEA The decision routine, in one line each

1. Predict an answer before reading the choices.
2. Eliminate with a named reason (Section 4.3), not a feeling.
3. On data and evidence questions, apply both filters: accurate *and* relevant.
4. If two remain, compare them at the exact words where they differ.
5. Rate your confidence; mark anything below "sure".

Execute and Check: Reliability

9 minutes Goal: make right ideas produce right answers, every time

5.1 Execution habits

Strong students lose points in execution not because they cannot do the algebra, but because they do it in their heads, fast, in several steps at once. A few habits remove most slips:

- **Organized scratch paper.** Number each problem. Write equations, not loose numbers. Leave space so you can find your work again in the review pass.
- **One risky operation per line.** Distributing a negative sign, squaring both sides, clearing fractions and moving terms across the equals sign are where slips happen. Give each its own line.
- **Units stay attached** until the end. Minutes and hours, percent and decimal, inches and feet.
- **Copy carefully.** Transcription errors (a dropped exponent, $+$ read as $-$, 18 copied as 81) are common at the top of the scale. Re-read the screen after copying anything with more than a few symbols.
- **Click what you mean.** After selecting a choice, look at it once more. Clicking the wrong letter is a real, recurring error.

5.2 Checking: methods ranked by strength

Not all checks are equal. Re-reading your own work is the weakest, because you see what you meant to write rather than what you wrote. From strongest to weakest:

1. **An independent route.** Solve a second way: Desmos instead of algebra, plugging in instead of deriving, an estimate instead of exact arithmetic. A different method will not repeat the same mistake.
2. **Substitute back into the original.** Put your answer into the problem as written on the screen, not into your rewritten equation. This also catches extraneous solutions from squaring.
3. **Re-read the question and compare with your boxed target.** Right quantity? Right units? Right direction?
4. **Reasonableness.** Is the sign right? Is the size plausible? Does it fit the story (a negative number of people, a probability above 1, a speed of 4,000 miles per hour)?

EXAMPLE the check that catches an extraneous root

Problem. What is the solution to $\sqrt{x+7} = x-5$?

Solution. Square both sides: $x+7 = x^2 - 10x + 25$, so $x^2 - 11x + 18 = 0$ and $(x-2)(x-9) = 0$. Check both in the *original*: $x = 9$ gives $\sqrt{16} = 4$ and $9 - 5 = 4$, which works. $x = 2$ gives $\sqrt{9} = 3$ but $2 - 5 = -3$, which fails. So the only solution is $x = 9$, and the choice that offers both 2 and 9 is the trap. Squaring can create false solutions; only substitution into the original finds them.

5.3 Checking in Reading and Writing

- **Reading questions:** point to the exact words in the passage that justify your choice. If you cannot point, you are guessing.

- **Grammar questions:** read the whole sentence with your choice in place, not just the few words around the blank. Many errors are only visible at full-sentence scale.
- **Transitions:** say the relationship between the two sentences in one word (contrast, cause, example, addition) and confirm your choice expresses that relationship.
- **Notes questions:** re-read the goal and check that *every part* of it is accomplished, not just that the choice uses the right signal word.

5.4 Time as a budget: the two-pass plan

Time is a resource you allocate. Spending three minutes on one question is a decision to spend less on others, so make that decision on purpose.

	Math module (35 min, 22 questions)	Reading and Writing module (32 min, 27 questions)
First pass	Aim to reach the last question with about 10 minutes left.	Aim to reach the last question with about 5 minutes left.
Skip rule	No real progress after about 90 seconds: make a best guess, mark it, move on.	Stuck between two choices after about 75 seconds: pick one, mark it, move on.
Second pass	Return to marked questions. Then use leftover time on independent-route checks of answers you rated “likely”.	Return to marked questions. Re-check any direction-flip or notes questions.
Pacing hints	Easy linear and percent questions should take 30 to 45 seconds. Bank that time for the hard ones.	Grammar and transitions: 20 to 40 seconds each. Bank the time for evidence, inference and two-text questions.

In each Reading and Writing module, the reading questions come first and the grammar, transition and notes questions come later. Because you can move freely within a module, some students start with the fast rule-based questions at the end and then return to the reading questions. Try both orders in timed practice and keep the one that gives fewer misses.

KEY IDEA Module 1 is different

Module 1 decides which Module 2 you get, and the top scores require the harder one. Do not rush Module 1 to save time; you cannot carry leftover time into Module 2. Use all of it to check. A careless miss in Module 1 costs you that point and may also lower your ceiling.

5.5 The error taxonomy

Every miss has a cause, and different causes need different fixes. Code every miss in your practice with one letter, and look at the counts every week. They tell you what to train.

Code	What happened	Countermeasure
R	Read. Answered a different question, misread a number or a logic word.	Box the target; highlight logic words; re-read the question before clicking.
T	Trap. Chose a designed wrong answer.	Predict first; name a reason for each elimination; two filters.
S	Slip. Arithmetic, sign, copying or clicking error.	One risky step per line; independent-route check.
K	Knowledge. Did not know a rule, fact, formula or word.	Add it to your toolkit sheet; drill it until automatic (Part 6).
P	Plan. Did not see how to start, or took a slow route.	Study the first move of the solution; add the pattern to your library (3.12).
C	Clock. Rushed or ran out of time.	Two-pass plan; skip rule; faster routes for easy questions.
F	Focus. Attention drifted: fatigue, a distracting topic, worry about the score.	Reset routine (5.6); sleep; full-length practice for stamina.

If you are already above about 1450, expect R, T and S to make up most of your list. That is good news, because those are the cheapest to fix.

5.6 Attention is a resource

A 2-hour-14-minute test is long enough for attention to fade. Learn the signs that you have drifted: reading the same line twice, thinking about the topic of a passage instead of the question, mentally adding up your score, or noticing a personal connection to the content. When you catch it, use a reset: put the pencil down, take one slow breath, and re-read only the question. It costs five seconds.

Attention also depends on things outside the test room. In the last week before the test, a fixed sleep schedule will do more for your score than one more practice set. Use the 10-minute break: stand up, eat something, and do not review your Reading and Writing performance in your head.

Tools You Should Not Have to Think About

8 minutes Goal: make the facts automatic so all your attention goes to the problem

Problem solving runs on a stock of facts. If you have to reconstruct the vertex formula or the semicolon rule during the test, that effort comes out of the attention you need for reading and checking. Everything on these pages should become automatic. If an item is not automatic yet, mark it and drill it.

6.1 Math: algebra and functions

TOOLKIT Lines, systems, functions

Lines. Slope $m = \frac{y_2 - y_1}{x_2 - x_1}$. Forms: $y = mx + b$; $y - y_1 = m(x - x_1)$; $Ax + By = C$ has slope $-\frac{A}{B}$.
Parallel lines: equal slopes. Perpendicular: slopes multiply to -1 .

Meaning in context. Slope is a rate (units of y per unit of x). The y -intercept is the starting value, when $x = 0$.

Systems. For $ax + by = c$ and $dx + ey = f$: no solution when $\frac{a}{d} = \frac{b}{e} \neq \frac{c}{f}$; infinitely many when $\frac{a}{d} = \frac{b}{e} = \frac{c}{f}$; exactly one otherwise.

Function notation. $f(a)$ means input a . $f(g(x))$: apply g first. " $f(x) = 5$ " asks for the input, not the output.

Transformations. $f(x) + k$: up k . $f(x - h)$: right h . $-f(x)$: flip over the x -axis. $f(-x)$: flip over the y -axis. $af(x)$: vertical stretch.

Absolute value. $|x - a| < b$ means $a - b < x < a + b$: the points within b of a .

Inequalities. Multiplying or dividing by a negative flips the inequality sign.

TOOLKIT Quadratics, polynomials, exponentials

Three forms of a quadratic. Standard $ax^2 + bx + c$: the y -intercept is c . Vertex $a(x - h)^2 + k$: the vertex is (h, k) . Factored $a(x - r)(x - s)$: the zeros are r and s . Choose the form that displays what the question asks for.

Vertex. $x = -\frac{b}{2a}$. The vertex is midway between the zeros.

Discriminant. $b^2 - 4ac$ positive: two real solutions; zero: one; negative: none.

Sum and product of roots. For $ax^2 + bx + c = 0$: sum $= -\frac{b}{a}$, product $= \frac{c}{a}$. Fast for “what is the sum of the solutions”.

Quadratic formula. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. It is not on the reference sheet.

Factor and remainder theorems. $(x - a)$ is a factor of $p(x)$ exactly when $p(a) = 0$. The remainder of $p(x) \div (x - a)$ is $p(a)$.

Special products. $(a \pm b)^2 = a^2 \pm 2ab + b^2$; $a^2 - b^2 = (a - b)(a + b)$.

Exponent rules. $x^a x^b = x^{a+b}$; $(x^a)^b = x^{ab}$; $x^{-a} = \frac{1}{x^a}$; $x^{m/n} = \sqrt[n]{x^m}$.

Exponential models. $y = a \cdot b^t$: a is the start, b the growth factor. Growth of $r\%$ per period means $b = 1 + \frac{r}{100}$; decay means $b = 1 - \frac{r}{100}$. “Doubles every 3 years”: $a \cdot 2^{t/3}$. Converting periods: a monthly factor m gives a yearly factor m^{12} .

Equations with radicals or variables in denominators. Always substitute solutions back; squaring and clearing denominators can create false ones.

6.2 Math: data, percents and geometry

TOOLKIT Percents, rates, statistics, probability

Percent change. $\frac{\text{new}-\text{old}}{\text{old}} \times 100$. Successive changes multiply: up 20% then down 20% is $1.2 \times 0.8 = 0.96$, a 4% loss.

Reverse percent. If \$84 is the price after a 30% discount, the original is $84/0.7 = \$120$, not 84×1.3 .

Units. Multiply by conversion factors written as fractions so that units cancel: $\frac{60 \text{ mi}}{1 \text{ h}} \times \frac{1 \text{ h}}{60 \text{ min}} = 1 \text{ mile per minute}$.

Center and spread. An outlier moves the mean but barely moves the median. Adding a constant to every value shifts the mean and median but not the standard deviation or range. Standard deviation measures how spread out values are around the mean.

Samples. A random sample supports conclusions about the population it was drawn from, and only that population. Random *assignment* to groups supports cause-and-effect conclusions. A larger sample gives a smaller margin of error.

Margin of error. An estimate of 42% with a margin of 3% means the plausible range is 39% to 45%. It does not mean every value in the range is equally likely, and it says nothing about individuals.

Conditional probability. $P(A \text{ given } B) = \frac{\text{number in both } A \text{ and } B}{\text{number in } B}$. The denominator is the group you are selecting from.

TOOLKIT Geometry and trigonometry

On the reference sheet. Areas of a circle, rectangle and triangle; circumference; the Pythagorean theorem; the 30-60-90 and 45-45-90 triangles; volumes of a box, cylinder, sphere, cone and pyramid; 360 degrees or 2π radians in a circle; 180 degrees in a triangle. Know what is there so you look it up instead of trying to remember it.

Not on the sheet. Slope, distance and midpoint, the quadratic formula, vertex form, the circle equation, exponent rules, and SOH-CAH-TOA.

Angles. A straight line is 180° . Parallel lines cut by a transversal make equal corresponding and alternate angles. The interior angles of an n -sided polygon sum to $(n - 2) \cdot 180^\circ$.

Similarity. Similar figures with length ratio k have area ratio k^2 and volume ratio k^3 . Look for similar triangles wherever there are parallel lines or a shared angle.

Pythagorean triples. 3-4-5, 5-12-13, 8-15-17, 7-24-25, and their multiples (such as 6-8-10).

Circles. $(x - h)^2 + (y - k)^2 = r^2$ has center (h, k) and radius r . If the equation is expanded, complete the square in x and in y . Arc length = $\frac{\theta}{360} \cdot 2\pi r$; sector area = $\frac{\theta}{360} \cdot \pi r^2$. In radians, arc length = $r\theta$. A tangent line is perpendicular to the radius at the point of tangency.

Trigonometry. $\sin \theta = \frac{\text{opp}}{\text{hyp}}$, $\cos \theta = \frac{\text{adj}}{\text{hyp}}$, $\tan \theta = \frac{\text{opp}}{\text{adj}}$. In a right triangle, $\sin x^\circ = \cos(90 - x)^\circ$. π radians = 180° .

6.3 Grammar: the rules behind conventions questions

TOOLKIT Sentence boundaries

Independent clause (IC). A subject plus a main verb that could stand alone as a sentence.

Legal ways to join two ICs. IC. IC IC; IC IC, and IC (or another of *for, and, nor, but, or, yet, so*) IC: IC (when the second explains or illustrates the first).

Illegal. IC, IC (a comma splice). IC IC with no punctuation (a run-on).

Conjunctive adverbs. *However, therefore, moreover, consequently, nevertheless* cannot join clauses with a comma alone. Use IC; however, IC or IC. However, IC.

Colons. What comes before a colon must be a complete sentence. What comes after can be a list, a phrase or a clause.

Extra information. A nonessential phrase is set off by a matching pair: two commas, two dashes, or two parentheses. Never mix a comma with a dash.

Essential information. No commas around information that identifies which one you mean: “the novelist Toni Morrison”, “the student who won”.

Never a single comma between a subject and its verb, between a verb and its object, or between a preposition and its object.

Lists. Commas between items; semicolons between items when the items already contain commas.

TOOLKIT Form, structure and sense

Every sentence needs a finite main verb. Words ending in *-ing*, *to* + verb, and *having* + verb cannot be the main verb on their own. If a choice leaves the subject without a real verb, the sentence is a fragment. Strip the sentence to its skeleton to check.

Subject-verb agreement. Find the true subject by crossing out prepositional phrases and interrupters. *Each*, *every*, *either*, *neither* and most collective nouns (*team*, *collection*, *group*) are singular.

Pronouns. Match number: *it/its* for a singular noun such as *the company*, *they/their* for plural. *Its* is possessive; *it's* means *it is*.

Verb tense. Match the time markers (*in 1850*, *currently*, *by next year*) and the tense of the surrounding sentences.

Modifiers. An opening descriptive phrase must be followed directly by the noun it describes. “Built in 1890, *the bridge...*”, not “Built in 1890, *engineers...*”.

Possessives and plurals. *scientist's* (one), *scientists'* (several), *scientists* (plural, not possessive).

6.4 Transitions and notes questions

TOOLKIT Transitions

Method. Read the sentence before the blank and the sentence after it. Name their relationship in one word, then choose the transition from that category.

Contrast. however, by contrast, on the other hand, yet, still, instead

Concession. nevertheless, even so, granted, admittedly

Addition. moreover, furthermore, additionally, also

Similarity. similarly, likewise

Cause and effect. therefore, thus, consequently, as a result, hence

Example. for example, for instance, specifically

Sequence. first, then, subsequently, later, finally, meanwhile

Clarification or emphasis. in other words, that is, in fact, indeed

Summary. in short, overall, ultimately

USE WHEN the blank is at the start of a sentence or clause. If two transitions from the same category both fit, the question is usually testing a finer difference (contrast versus concession, for instance).

TOOLKIT Notes questions (rhetorical synthesis)

Read the goal first. The question tells you what the sentence must do: emphasize a difference, introduce a study to an unfamiliar audience, present a finding, explain a method. Write the goal in two or three words.

Test each choice against the goal only. All four choices are usually accurate to the notes. Accuracy is not the test; the goal is.

Watch for signal words that hide the content. A choice can contain “differ”, “unlike” or “both” and still fail to state an actual difference or similarity. Check what the sentence *says*, not which words it uses.

Audience matters. “For an audience unfamiliar with X” means the right choice explains what X is. “For an audience already familiar” means it does not need to.

Every part counts. If the goal has two parts (“present the study and its conclusion”), the right choice does both.

6.5 Reading question types: a first move for each

Question type	First move
Words in context	Cover the choices. Put your own word in the blank based on the sentence's logic, then match. Watch for common words used in a less common sense.
Structure and purpose	Describe what the sentence or text <i>does</i> (introduces, contrasts, qualifies, illustrates) before reading the choices.
Two texts	One line for each author's position. Find the exact point where they touch; how would author 2 respond to that specific point? Do not assume total disagreement.
Main idea	The claim the passage is built to support, not the background and not a single detail.
Textual evidence	State the claim and the direction (support or weaken). For each choice ask: if this were true, would the claim become more likely, less likely, or unaffected?
Data	Read the title, the axis labels, the units and the legend first. Then apply both filters: accurate and relevant.
Logical completion	Follow the chain of reasoning to the end. The answer is the narrowest conclusion that must follow, not an interesting extra claim.

Training to 1600

6 minutes Goal: turn practice into fewer misses, not just more questions done

7.1 How problem-solving skill grows

Doing many questions is not the same as getting better at them. Skill grows from **deliberate practice**: problems at the edge of your ability, fast feedback, and honest reflection on your *process*, not just your answer. A student who does 40 hard questions and reviews each miss for five minutes improves faster than one who does 200 and checks only the score. The review is where the learning happens.

7.2 The review protocol

Apply this to every miss **and** every question you got right while unsure. A lucky guess is a miss you have not had yet.

1. **Re-solve it cold**, without looking at the explanation. Often you will get it right, which tells you the cause was R, T, S, C or F, not K.
2. **Code the error** with one letter from Section 5.5.
3. **Write the first move** or the rule in one line: “Exactly one solution means discriminant zero.”
4. **Explain the trap**: why did the wrong answer attract you? Name its design (Section 4.3).
5. **Write a trigger rule**: “When I see __, I will __.” For example: “When I see ‘given that’, I will circle the row or column first.”
6. **Revisit** the question after 3 days and again after 2 weeks. If you miss it again, the trigger rule is not working yet; rewrite it.

7.3 An error log you can copy

Source	Skill	Code	First move or rule	Trigger rule: when I see..., I will...	Redo dates

Count the codes each week. The largest count is your next training target, whatever topic it happens to fall in.

7.4 A weekly plan for the last stretch

For a student scoring roughly 1450 to 1550 who wants 1600:

- **One full practice test** under real conditions in Bluebook: timed, in one sitting, with the break, on the device you will use. Review it the same day or the next; the review should take at least as long as the test.
- **Three or four targeted sessions** of 45 to 60 minutes, drawn from your error counts: timed sets of 10 hard questions in your weakest skills, each followed by review.

- **One error-log session:** redo the questions due for revisiting.
- **Ten minutes a day** of something automatic: grammar rules, transitions, a Desmos drill, or a page of Part 6.

7.5 Official material

- **Bluebook practice tests** (free, in the Bluebook app) are the closest thing to the real test, including the adaptive modules and the tools. Save two or three unseen tests for the final weeks.
- **The SAT Suite Question Bank** (free, on the College Board website) lets you filter questions by section, domain, skill and difficulty. Filtering to hard questions in one skill is the most efficient targeted practice available.
- Other publishers' material is useful for volume, but its difficulty and style vary. When it disagrees with official material, trust the official material.

7.6 The last two weeks and test day

- **Two weeks out:** fix your sleep schedule to match the test-day wake-up time. Take your last unseen full tests.
- **Final two days:** no new content. Re-read your trigger rules and Part 6. Do a few easy sets to stay warm.
- **Night before:** pack your device with Bluebook updated and fully charged, the charger, your admission ticket, photo ID, pencils, and a snack and water for the break. Check the College Board's own test-day checklist for anything that has changed.
- **During the test:** run the routine. Target, conditions, plain words. Predict, eliminate with reasons, check by an independent route. Mark anything below "sure". Use the break to reset, not to replay the first section.

KEY IDEA The whole packet in six lines

1. Box the target and highlight the logic words before you start.
2. Choose the representation that makes the problem easy: equation, graph, table, picture, or plain words.
3. When stuck, run the heuristic list; you always have a next move.
4. Predict, then eliminate with named reasons. Data and evidence answers must be accurate *and* relevant.
5. Check by a different route than the one you used to solve.
6. Log every miss by cause, and train the cause, not just the topic.

Practice Set

about 30 minutes to solve, 15 to review

Goal: use the method on fresh problems, then check it against the key

Each problem is tagged with the idea it practices. Work them in order, without a time limit the first time. For each one, write down the target before you start and your confidence level (sure, likely, guess) when you finish. Then check the key and apply the review protocol from Section 7.2 to every miss and every unsure answer. Calculator allowed for all math.

8.1 Math

M1 **structure** If $5x - 3y = 12$, what is the value of $10x - 6y + 3$?

M2 **definitions** The function p is defined by $p(x) = x^3 + ax^2 - 4x - 12$, where a is a constant. If $p(-2) = 0$, what is the value of a ?

M3 **definitions** The equation $x^2 - 6x + k = 2x - 7$ has exactly one real solution. What is the value of k ?

M4 **definitions** The system $3x + ky = 12$ and $9x + 15y = c$, where k and c are constants, has infinitely many solutions. What is the value of $k + c$?

M5 **plug in numbers** The price of a bicycle was increased by 25%. Later, the new price was decreased by $p\%$, and the final price was 10% less than the original price. What is the value of p ?

- A. 15
- B. 28
- C. 32
- D. 35

M6 **find what does not change** A chemist has 60 milliliters of a solution that is 30% acid. How many milliliters of pure water must be added to make a solution that is 20% acid?

- M7** tables, conditional probability A study recorded whether each of 250 adults owns a dog and whether each exercises daily.

	Exercises daily	Does not	Total
Owens a dog	60	40	100
Does not own a dog	55	95	150
Total	115	135	250

If one of the adults who exercises daily is selected at random, what is the probability that the adult owns a dog?

- M8** representation The graph of $x^2 + y^2 - 10x + 4y = 7$ in the xy -plane is a circle. What is the radius of the circle?

- M9** extremes A theater's daily revenue R , in dollars, from selling tickets at p dollars each is modeled by $R(p) = -20p^2 + 800p$. What is the greatest daily revenue the model predicts?

- M10** tools: sum of roots What is the sum of the solutions to $2x^2 - 7x - 15 = 0$?

- M11** tools: similarity Two cylinders are similar, and the ratio of their volumes is $8 : 27$. What is the ratio of their surface areas?

- A. $2 : 3$
- B. $4 : 9$
- C. $8 : 27$
- D. $16 : 81$

- M12** check in the original What is the solution set of $\sqrt{2x + 9} = x + 3$?

- A. $\{-4\}$
- B. $\{0\}$
- C. $\{-4, 0\}$
- D. no solution

-
- M13** **translation** The volume V , in liters, of water in a tank t minutes after it begins draining is $V(t) = 900 - 15t$. What is the best interpretation of 15 in this context?
- A. The tank holds 15 liters when it is full.
 - B. The water drains at a rate of 15 liters per minute.
 - C. The tank takes 15 minutes to drain completely.
 - D. The tank holds 15 liters after 1 minute of draining.

8.2 Reading and Writing

- R1** **predict first** The committee's report was notably ———: it described the budget shortfall but offered no recommendation about which programs to cut, leaving that decision entirely to the council.
- Which choice completes the text with the most logical and precise word or phrase?
- A. noncommittal
 - B. exhaustive
 - C. decisive
 - D. inaccurate

-
- R2** **role map** For decades, astronomers assumed that the outer solar system was too cold and dark to hold liquid water. Data from spacecraft have since complicated that view. Measurements of Jupiter's moon Europa, for example, suggest that a salty ocean lies beneath its icy crust, kept liquid by heat generated as Jupiter's gravity flexes the moon's interior.
- Which choice best describes the overall structure of the text?
- A. It presents a long-held assumption, then offers evidence that challenges it.
 - B. It describes a scientific debate, then explains why the debate cannot be resolved.
 - C. It introduces a hypothesis about Europa, then rejects it on the basis of new data.
 - D. It compares two of Jupiter's moons, then explains how they differ.

R3 **direction flip** Biologists studying songbirds have proposed that birds in cities sing at a higher pitch than rural birds of the same species because higher-pitched songs are easier to hear over the low rumble of traffic.

Which finding, if true, would most directly weaken the biologists' proposal?

- A. Urban songbirds have shorter breeding seasons than rural songbirds of the same species.
- B. Songbirds in a large city where motor vehicles have been banned for over a century sing at the same higher pitch as songbirds in cities with heavy traffic.
- C. Songbirds that nest near busy highways in rural areas sing at higher pitches than rural songbirds of the same species that nest far from roads.
- D. Some urban songbird species sing more loudly than rural members of the same species.

R4 **narrowest conclusion** Archaeologists found grinding stones at a 12,000-year-old site. Microscopic residue on the stones came from wild barley, not from domesticated barley, which first appears in the region's archaeological record about 1,500 years later. Because grinding stones are used to process grain into flour, the find suggests that _____

Which choice most logically completes the text?

- A. people at the site were processing wild grain before barley was domesticated.
- B. domesticated barley was first developed at this site.
- C. grinding stones were invented in order to process domesticated barley.
- D. wild barley was more nutritious than domesticated barley.

R5 **tools: boundaries** The volcano had been quiet for nearly four _____ in 2011, a series of small earthquakes signaled that magma was rising beneath it.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A. centuries, however,
- B. centuries; however,
- C. centuries however,
- D. centuries, however

-
- R6** **skeleton** The research buoy, anchored in the North Atlantic in 1998 and battered by decades of storms, ——— temperature readings to the institute every hour.

Which choice completes the text so that it conforms to the conventions of Standard English?

- A. transmitting
- B. to transmit
- C. having transmitted
- D. transmits

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- R7** **name the relationship** Most substances contract as they freeze, so their solid forms are denser than their liquid forms. Water is an exception: it expands as it freezes. ——— ice floats, forming a layer on lakes that insulates the water beneath it during winter.

Which choice completes the text with the most logical transition?

- A. For example,
- B. Nevertheless,
- C. As a result,
- D. Similarly,

-
- R8** **goal first** While researching a topic, a student has taken the following notes:

- The Arden Seed Bank stores seeds of about 4,000 wild plant species.
- It keeps its seeds at -20°C to slow their aging.
- The Merrow Seed Bank stores seeds of about 2,500 crop varieties.
- It also keeps its seeds at -20°C .
- The two seed banks share duplicate samples with each other as a backup.

The student wants to emphasize a difference between the two seed banks. Which choice most effectively uses relevant information from the notes to accomplish this goal?

- A. Both the Arden and Merrow seed banks keep their seeds at -20°C , and they share duplicate samples.
- B. While the Arden Seed Bank stores seeds of wild plant species, the Merrow Seed Bank stores seeds of crop varieties.
- C. Unlike many seed banks, the Arden Seed Bank keeps its seeds at -20°C to slow their aging.
- D. The Arden and Merrow seed banks differ in that they share duplicate samples with each other as a backup.

R9 two one-liners **Text 1:** Many historians have argued that the printing press spread literacy across Europe quickly, because cheaper books gave ordinary people a reason to learn to read.

Text 2: Records from parish schools suggest a slower story. In many regions, literacy rates barely changed for a century after printing arrived; they rose sharply only once local schools became common. Cheap books, it seems, mattered less than having someone to teach reading.

Based on the texts, how would the author of Text 2 most likely respond to the historians' argument in Text 1?

- A. By agreeing that cheaper books were the main reason literacy spread quickly
- B. By arguing that the spread of literacy depended more on access to schooling than on the availability of cheap books
- C. By claiming that the printing press had no effect on European society
- D. By suggesting that parish school records are unreliable sources of information about literacy

8.3 Answer key with explanations

M1 **27**
Structure: $10x - 6y = 2(5x - 3y) = 24$, so the value is $24 + 3 = 27$. Do not solve for x and y ; you cannot, and you do not need to.

M2 **3**
Definition: $p(-2) = -8 + 4a + 8 - 12 = 4a - 12 = 0$, so $a = 3$. (Then $p(x) = (x + 3)(x - 2)(x + 2)$.)

M3 **9**
Move everything to one side: $x^2 - 8x + (k + 7) = 0$. Exactly one solution means the discriminant is zero: $64 - 4(k + 7) = 0$, so $k + 7 = 16$ and $k = 9$. Common slip: forgetting to move $2x - 7$ before using the discriminant. Check with Desmos: graph both sides with a slider for k .

M4 **41**
Infinitely many solutions means the second equation is a multiple of the first. Since $9 = 3 \times 3$, the multiplier is 3: $15 = 3k$ gives $k = 5$, and $c = 3 \times 12 = 36$. So $k + c = 41$.

M5 **(B) 28**
Plug in \$100: after the increase, \$125. The final price is \$90, so the decrease is \$35, which is $\frac{35}{125} = 28\%$ of \$125. Trap (D) adds the percents ($25 + 10$); trap (A) subtracts them.

M6 30

The acid is fixed: $0.30 \times 60 = 18$ mL. At 20%, the total must be $18/0.20 = 90$ mL, so add $90 - 60 = 30$ mL of water.

M7 $\frac{12}{23} \approx 0.5217$

Select only from the 115 adults who exercise daily, so the probability is $\frac{60}{115} = \frac{12}{23}$. Traps: $\frac{60}{100} = 0.6$ (the dog row) and $\frac{60}{250} = 0.24$ (everyone). Either $12/23$ or $.5217$ is an acceptable typed answer.

M8 6

Complete the square: $(x^2 - 10x + 25) + (y^2 + 4y + 4) = 7 + 25 + 4$, so $(x - 5)^2 + (y + 2)^2 = 36$. The radius is $\sqrt{36} = 6$. Trap: answering 36, which is r^2 .

M9 8000

The vertex is at $p = -\frac{800}{2(-20)} = 20$, and $R(20) = -8000 + 16000 = 8000$. Trap: answering 20, the price that gives the maximum, rather than the maximum itself. Check the boxed target.

M10 $\frac{7}{2}$ or 3.5

Sum of roots $= -\frac{b}{a} = \frac{7}{2}$. Check by factoring: $(2x + 3)(x - 5) = 0$ gives $-\frac{3}{2}$ and 5, which sum to $\frac{7}{2}$.

M11 (B) 4 : 9

The volume ratio is $k^3 = \frac{8}{27}$, so the length ratio is $k = \frac{2}{3}$ and the area ratio is $k^2 = \frac{4}{9}$.

M12 (B) {0}

Squaring gives $2x + 9 = x^2 + 6x + 9$, so $x^2 + 4x = 0$ and $x = 0$ or $x = -4$. Check in the original: $x = 0$ gives $3 = 3$; $x = -4$ gives $\sqrt{1} = 1$ but $-4 + 3 = -1$, so it fails. Trap (C) skips the check.

M13 (B)

15 is the coefficient of t , so it is a rate: liters lost per minute. (A) confuses it with the starting value 900. (C) would be $900/15 = 60$ minutes. (D) would be 885 liters.

R1 (A) noncommittal

Prediction from the colon clause: "did not take a position". *Noncommittal* matches. (B) is not supported: the report "described" the shortfall, with no sign of thoroughness. (C) is the opposite. (D) is never suggested.

R2 (A)

Role map: “old view”, “but”, “example”. (B): there is no claim that anything cannot be resolved. (C) reverses it: the old assumption is challenged, and the Europa evidence is the challenge. (D): only one moon is discussed.

R3 (B)

Plain words: “which fact shows traffic noise is not the cause?” High pitch in a city with no traffic means something other than traffic noise produces it. (C) is the direction trap: it *supports* the proposal. (A) is irrelevant to pitch. (D) is about loudness, not pitch.

R4 (A)

Follow the chain: grinding stones process grain into flour; the residue is wild barley; domestication came 1,500 years later. The narrowest conclusion: wild grain was processed before domestication. (B) and (C) contradict the timeline. (D) is out of scope.

R5 (B)

Both sides are independent clauses, and *however* is a conjunctive adverb, so it needs a semicolon (or a period) before it. (A) and (D) are comma splices. (C) is a run-on.

R6 (D) transmits

Skeleton: *The research buoy* [...] ——— readings. The subject has no main verb yet, so the blank must supply a finite verb. (A), (B) and (C) are non-finite forms and leave a fragment.

R7 (C) As a result,

Relationship: water expands when it freezes, so ice is less dense, so it floats. That is cause and effect. (A) treats it as an example, (B) as a contrast, (D) as a parallel.

R8 (B)

Goal: a difference between the two banks. Only (B) states one (wild species versus crop varieties). (A) states similarities. (C) uses “unlike” but compares Arden with unnamed other banks and brings in information not in the notes. (D) uses “differ” but states something the two banks have in common. Signal words are not content.

R9 (B)

One-liners: Text 1, “cheap books spread literacy quickly”; Text 2, “schools mattered more than books”. The point of contact is the cause of literacy’s spread. (A) is the opposite of Text 2. (C) is too strong: Text 2 says books mattered *less*, not that printing had no effect. (D) reverses Text 2, which relies on those records.

KEY IDEA Scoring yourself

Count your misses, but also count your unsure correct answers. For a student aiming at 1600, the useful number is *misses plus unsure answers*. When that number is close to zero on fresh hard material, under time pressure, you are ready.